Phonon Absorption at the Magnetoroton Minimum in the Fractional Quantum Hall Effect

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We have made the first phonon absorption measurements in the fractional quantum Hall regime. Experiments have been conducted on two samples which have similar electron densities but greatly differing mobilities. The energy gaps as measured by activation studies of the longitudinal resistance differ by a factor of 2. Phonon absorption measurements give almost identical values for the energy gap, demonstrating that the gap measured in this way is rather insensitive to disorder. The value of this gap is in agreement with the activation gap measured in the high mobility sample. Values obtained at $\nu = 2/3$ are in good agreement with theory.

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The fractional quantum Hall effect (FQHE), which occurs in a high mobility two-dimensional electron system (2DES) subject to a strong perpendicular magnetic field, is ascribed to the existence of an incompressible quantum liquid at certain rational filling factors. It is believed that the low lying excited states of the liquid are collective modes which are never gapless (except at the sample boundaries). Girvin, MacDonald, and Platzman (GMP) [1] have developed a theory for these collective modes at the primary filling factors $\nu = 2\pi l_c^2 n_s = 1/m$ (n_s is the electron sheet density, $l_c = \sqrt{\hbar/eB}$ is the cyclotron length, and m is an odd integer); they find that the dispersion of the collective mode has a deep minimum for wavelengths comparable to the mean interparticle spacing. This "magnetoroton" minimum occurs, as in liquid helium, because of a peak in the static structure factor. Recently a new (but presumably equivalent) framework for understanding the FQHE has emerged [2,3] based on the idea that the fractional quantum Hall effect of electrons is equivalent to the integer quantum Hall effect of composite fermions consisting of charges bound to point fluxes. The theory of the collective excitations within this framework is currently being developed [4,5] but is not yet in quantitative agreement with the calculation of [1]. What has so far been lacking is any experimental measurement of the gap close to the magnetoroton minimum. We report on the first such measurements.

Experimental studies of energy gaps in the FQHE state have been pursued in several ways including activated magnetotransport measurements where disorder is known to affect the values obtained [6]. Pinczuk *et al.* [7] observe a feature in the inelastic light scattering spectrum that is attributed to the low wave-vector excitations of the FQHE state in a quantum well. Photoluminescence experiments find anomalies in the spectra in the FQHE regime [8], but quantitative interpretation of these results requires a detailed understanding of the dynamical response of the 2DES in optical recombination processes.

Ballistic acoustic phonons have proved to be a unique probe of low-dimensional systems [9]. The typical energies and wave vectors are well matched to those of the 2DES and since, in the FQHE state, the magnetorotons are the only low energy modes which can couple to the ground state through the electron density, they should provide the principal channel for the absorption of acoustic phonons. For example, the dispersion curve of a longitudinal phonon incident on a 2DES in the FQHE state at an incident angle of 60° crosses the collective mode dispersion curve close to the magnetoroton minimum. This makes phonon absorption a promising method to investigate the energy gap of the FQHE in this region.

We have studied two high mobility GaAs-AlGaAs heterostructures with similar electron densities (1.50×10^{11} cm^{-2} for NU409 and 1.51×10^{11} cm^{-2} for G635) but greatly differing mobilities (1.0×10^6 cm^2 V^{-1} s^{-1} for NU409 and 8.0×10^6 cm^2 V^{-1} s^{-1} for G635). Both devices were formed from heterojunctions separated by a pure AlGaAs spacer layer from a silicon-doped AlGaAs layer. Both devices were illuminated with red light to give the carrier densities quoted. A 3 mm $\times$ 2 mm Hall bar was defined, the rear face of each sample was polished to an optical finish, and 600 μm $\times$ 60 μm constantan heaters evaporated opposite the Hall bar. The positions of the heaters were determined using front-to-back alignment. One heater was positioned opposite the middle of the Hall bar while the other was over an edge. The samples were mounted *in vacuo* on the tail of a dilution refrigerator.

The phonon absorption was measured from the change in longitudinal resistance produced by a burst of nonequilibrium ballistic phonons. The 2DES was supplied with a

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constant bias current, and the phonons were generated by applying a 100 ns electrical pulse to one of the constantan heaters. After the application of the heater pulse the phonon spectrum in the substrate has two peaks: the lower energy peak corresponding to the equilibrium temperature of the substrate before the pulse was applied and the higher energy one corresponding to the heater temperature. Eventually the phonon spectrum will equilibrate at a slightly raised substrate temperature. At the low temperatures used in this experiment the phonon mean free path is much greater than the size of the sample so that relaxation is dominated by diffuse scattering at the sample edges and at Ohmic contacts. Because the time for a phonon to cross the sample is about 100 ns, relaxation to equilibrium is a very slow process. The results reported here are concerned with the behavior of the electron system due to the nonequilibrium part of the spectrum rather than the heating of the substrate.

We have measured the time resolved response of the electron gas on G635 to the phonon pulse. The results demonstrate that the maximum electrical response occurs on a time scale consistent with the phonon time of flight. This leads us to believe that we are measuring the ballistic response. The electrical response decays with a time constant of the order of 2 μs. To obtain better signal to noise ratios than are possible with the time resolved measurements we used a gated amplifier to measure the increase in longitudinal resistance in the tail of the response. We assume this resistance change to be proportional to the power absorbed from the phonon pulse and have measured it as a function of heater temperature. All of the results reported here were made using the gated amplifier. At all times we were careful to check that the measured gap was independent of experimental parameters such as the bias current, the length and repetition rate of the heater pulses, the delay and width of the gate signal, and the substrate temperature (provided this was kept well below the gap temperature). In addition, although the magnitude of the response differed between the two heaters, the value of the gap did not.

The spectral distribution of the phonons was assumed to correspond to that of a blackbody at a temperature, T_h, which is calculated from the total power dissipated in the heater by acoustic mismatch theory [10] and confirmed to within 10% by measuring the energy gaps at $\nu = 12$ and $\nu = 14$ on G635 under the same conditions as the fractional energy state gaps were obtained.

We have also carried out measurements of the activated magnetoresistance at $\nu = 2/3$ over a range of temperatures and fitted the results to the usual form $R_{xx} \sim \exp[-\Delta_{\mathrm{tr}}/2k_B T]$ (with a correction for hopping at low temperatures). This leads to values for Δ_{tr} of 2.7 ± 0.2 and 5.5 ± 0.5 K for NU409 and G635, respectively.

The rate of energy transfer to the 2DES from the phonon beam can be expressed, using first order perturbation theory, as

$$P(T_h) = \sum_s \int \frac{d^3\underline{Q}}{(2\pi)^3} \hbar\omega_s(\underline{Q}) n_s(\underline{Q}) |M_s(\underline{Q})|^2 S(\mathbf{q}, \omega_s(\underline{Q})), \quad (1)$$

where $\underline{Q} = (\mathbf{q}, q_z)$ is a phonon wave vector, $\hbar\omega_s(\underline{Q}) = \hbar c_s Q$ is the energy of a phonon of wave vector $\underline{Q}$ and polarization s (= LA, TA_1, TA_2), $n_s(\underline{Q})$ is the number of phonons in the given mode, $M_s(\underline{Q})$ is the electron-phonon coupling, and $S(\mathbf{q}, \omega)$ is the dynamic structure factor of the 2DES. We assume that the phonon occupation is equal to the Bose-Einstein distribution for all $\underline{Q}$ vectors within the geometrical cone subtended by the 2DES at the heater. The single mode approximation of [1] amounts to assuming that the dynamic structure factor has the form

$$S(\mathbf{q}, \omega) = \bar{s}(\mathbf{q}) \delta(\omega - \Delta(\mathbf{q})), \quad (2)$$

where $\bar{s}(\mathbf{q})$ is the lowest Landau level projected *static* structure factor (obtained by GMP from the wave function of Laughlin [11]) and $\Delta(q)$ is the magnetoroton energy [expressed by GMP as a functional of $\bar{s}(\mathbf{q})$]. We assume that the results of [1] are qualitatively correct for all FQHE states, i.e., that $\Delta(q)$ has a gap for all q and has a minimum at some finite value q^*. For temperatures $T_h \ll \Delta(q^*)$ we can evaluate the integral in Eq. (1) by steepest descents leading to the result $P(T_h) \sim n_s(\underline{Q}) = \{\exp[\Delta(q^*)/k_B T_h] - 1\}^{-1}$. So, if we assume that the response, $\delta R_{xx}(T_h)$, to a heater pulse at constant bias current I_b satisfies $I_b^2 \delta R_{xx}(T_h) = P(T_h)$, then fitting the size of the voltage pulse to the Bose distribution will yield $\Delta(q^*)$. The variation of the response to the phonon pulse with inverse heater temperature for the $\nu = 2/3$ fraction is shown in Fig. 1 for both samples: the lines are the predicted variation using the best fit value of $\Delta(q^*)$; the points are experimental data. Since the scale of the voltage response cannot be absolutely predicted, it is used as a fitting parameter.

It is clear that, for all temperatures of interest, the absorption will be dominated by the magnetoroton minimum where the static structure factor is maximal, the gap is minimal, and the density of magnetoroton states is maximal. For the primary fraction $\nu = 2/3$ (equivalent to the $\nu = 1/3$ state by particle-hole symmetry) we can use the results of [1] to evaluate the integral numerically. We assume that the magnetoroton dispersion is simply that calculated by GMP, scaled by a q independent fitting parameter. This procedure yields a value for $\Delta(q^*)$ which is (just) within the experimental error of that obtained by fitting to the Bose distribution; all gaps quoted here are those obtained by the simpler method. One result of the more detailed calculation is that the primary channel for energy transfer is via the deformation potential coupling to the LA phonons; these are only weakly focused by the lattice, so we neglect phonon focusing effects entirely. More details of the calculations will be given elsewhere [12]. The results of these measurements are displayed in Table I.

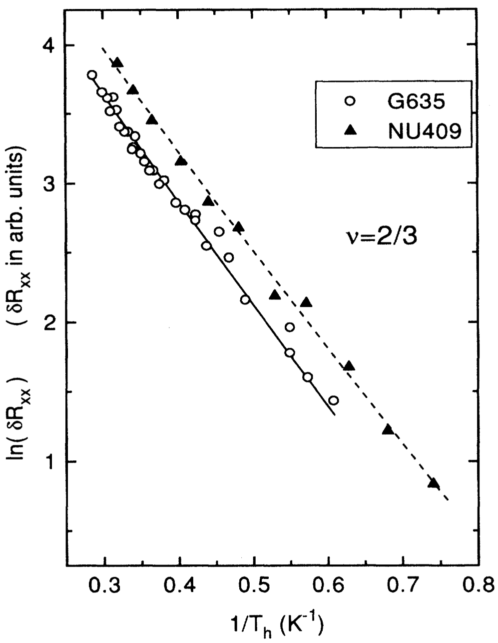

FIG. 1. Plot of the ballistic phonon response, $\ln(\delta R_{xx})$, against $1/T_h$ for both samples at $\nu = 2/3$.

The key point in our interpretation of the experiment is that we believe that we are observing changes in the dissipation of the 2DES which are a direct result of the creation of elementary excitations of the system when phonons are absorbed. An alternative scenario is that the 2DES is simply being heated by the phonon beam so that we are actually just measuring the activated magnetoresistance at some raised temperature T_e. The repetition rate of the heater pulse is kept low ($\sim$1 kHz) so that the average power transferred to the device is less than 20 nW for a heater temperature of less than 2 K. By comparing the longitudinal magnetoresistance at several substrate temperatures with and without heater pulses we estimate that the 20 nW from the heater pulse causes the substrate temperature to rise from 50 to about 80 mK. During our experiments the tail of the dilution refrigerator was temperature controlled to minimize the change in substrate temperature. However, the electron system could be raised to a higher temperature during the pulse. If the 2DES were simply being heated by the phonon beam, then its temperature should depend only on the phonon intensity, not on the spectral distribution. Evidence against the simple heating interpretation comes from the fact that the magnitude of the response to the two heaters is quite different, indicating that the phonon intensities actually incident on the 2DES due to the two heaters are very different, but the value of the gap extracted is the same, showing that the electrical response depends on the spectral distribution of the phonons, not on their intensity. Hence we believe that we are seeing the direct response of the 2DES to the ballistic phonons. A simple estimate of the heating effect also shows that the heating explanation is inconsistent with our results.

TABLE I. Magnetoroton energy gaps measured by phonon absorption.

ν	$\Delta(q^*)$ (K)	$\Delta(q^*)/(e^2/4\pi\epsilon\epsilon_0 l_c)$
NU409		
2/3	6.2 ± 0.2	0.041 ± 0.002
G635		
2/3	6.9 ± 0.4	0.045 ± 0.003
3/5	3.5 ± 0.3	0.021 ± 0.002
4/7	2.6 ± 0.4	0.014 ± 0.002

The energy gaps determined from the phonon experiment are nearly equal for the two samples, but those determined from activated transport measurements differ by a factor of just over 2, in line with the difference in their zero field mobilities. Clearly disorder affects the two measurements in different ways. We interpret this difference as being due to large scale inhomogeneity in the lower mobility sample leading to a variation in the local conduction band edge. The transport measurements are taken at thermal equilibrium so that the measured gap will be dominated by the largest negative fluctuation in the band edge. Since we conclude that the phonon experiment is a genuine local spectroscopic probe, measuring the excitation gap at a specific point in the 2DES, it should only be affected by short length scale disorder.

The energy gap obtained at $\nu = 2/3$ on G635 can be compared to the theoretically predicted value for $\nu = 1/3$. Two effects that must be accounted for are the finite thickness of the 2DES and Landau level mixing. In G635, at $\nu = 2/3$, $bl_c = 1.75$, where b is the Fang-Howard parameter. For this value of bl_c theoretical studies [13] predict that the energy gap is reduced by a factor of 0.7 from the ideal case of a 2DES with zero thickness. A rough estimate of the effect of Landau level mixing can be made following the work of Yoshioka [14]. When these two effects are combined, the theoretical gap is found to be $\Delta = ce^2/4\pi\epsilon\epsilon_0 l_c^2$, where $c \approx 0.04$, in good agreement with our experimentally obtained value of $c = 0.045 \pm 0.002$. The small difference between the NU409 and G635 may be due to the differences in layer structure and doping levels; this may alter the vertical extent of the wave function. Another possibility is that it is due to the higher disorder in NU409. Pinczuk *et al.* [7] have measured the excitation curve in the limit of low wave vector by inelastic light scattering in a quantum well. Allowing for the differences between the samples our results are in good agreement, assuming that $\Delta(0) \approx 2\Delta(q^*)$ [1].

The mechanism by which phonon absorption causes a change in the longitudinal voltage is as yet unknown. One possibility is that there is a mutual friction between the magnetorotons and the Laughlin liquid (possibly due to the dipole moment of the former) which causes dissipation but, as pointed out by Platzman [15], this would mean that magnetotransport experiments measure $\Delta_{tr} = 2\Delta(q^*)$ rather than Δ_∞. Another possibility is that once created by a high energy phonon, a magnetoroton can absorb many low energy phonons and so dissociate into an unbound quasiparticle and quasihole [12]. Research is in progress to determine which of these is the dominant effect.

There are no calculations analogous to that of GMP for filling factors other than $\nu = 1/m$. Recent calculations employing Jain's [2] composite fermion (CF) picture have been carried out by Simon and Halperin [5] using the random phase approximation modified to include renormalization of the CF effective mass. These indicate that the magnetoroton gap, Δ^*, should scale approximately as $\Delta\omega^*$: the CF cyclotron frequency, which is the same as the scaling expected for the charge gap Δ_∞. $\Delta\omega^* = e(B - B_{1/2})/m^*$ for the fractions $\nu = 1/2p + 1$, where $B_{1/2}$ is the field at which $\nu = 1/2$ and m^* is the CF effective mass. Hence one expects that the gaps measured by both techniques should scale linearly with $B - B_{1/2}$, with a slope inversely proportional to the CF effective mass (if this is constant, or indeed finite). Our results are shown in Fig. 2. The variation of the energy gaps at $\nu = 2/3$, 3/5, and 4/7 with magnetic field is consistent with the CF theory. The value of the CF effective mass is similar to that obtained by magnetotransport [16], at $m^* = (8 \pm 1)m_b$ where m_b is the band mass of the electrons in GaAs. Within the calculation of GMP the difference between Δ^* and Δ_∞ is of the order of 20%; owing to the difference in the way the disorder affects the two measurements it is hard to compare the two results to within this level of uncertainty. The energy gap falls to zero before $\nu = 1/2$, and the intercept at $\nu = 1/2$ is

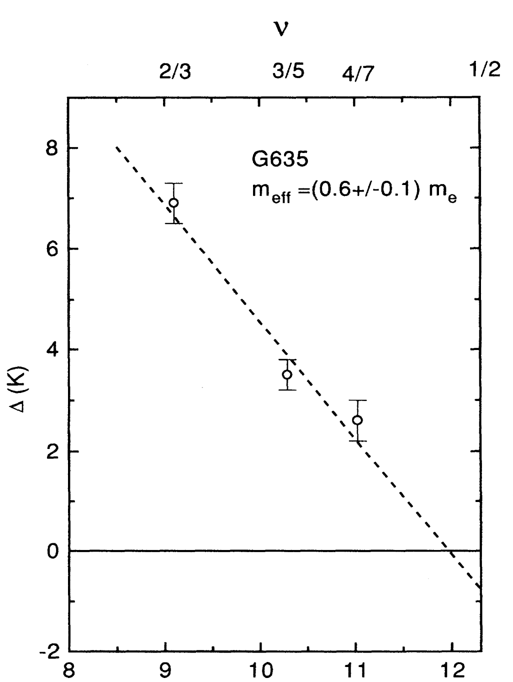

FIG. 2. Plot of energy gap $\Delta(q^*)$, determined by phonon absorption as a function of magnetic field.

-0.7 ± 0.5 K. The value of this intercept obtained from magnetotransport results on the same sample is ≈ -2 K; this effect has previously [16] been attributed to sample disorder. Allowing for measurement uncertainties the effect of disorder on the absorption experiments does not exceed 0.7 K. These observations support the argument that the absorption measurements are not affected by disorder as much as the magnetotransport results.

In conclusion, the energy of the magnetoroton minimum for several filling factors of a high mobility heterojunction has been measured by phonon absorption. The value at $\nu = 2/3$ is in excellent agreement with theory. The values for $\nu = 2/3$, 3/5, and 4/7 are consistent with the composite fermion theory of the FQHE although detailed comparison with theory has not yet been made.

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Note added.—If the data in Fig. 2 are plotted as $\Delta^*(\nu = p/2p + 1)$ vs $e^2/4\pi\epsilon l_c(2p + 1)$ rather than against $B - B_{1/2}$, then an equally good straight line fit is obtained, yielding an effective mass (at $\nu = 1/2$) of $m^* = (12 \pm 3)m_b$.

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